

Equilibrium Non-Panic Bank Failures*

Guilherme Carmona Patrick Leoni
Universidade Nova de Lisboa University of Minnesota

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Abstract

We observe many episodes in which a large number of people attempt to withdraw their deposits from a bank, forcing it to suspend withdrawals or even to fail. In contrast with the view that those episodes are driven by consumers' panic or sunspots, we propose to explain them as a consequence of the conjunction of lack of full back up of deposits by banks, and of an unexpectedly high fraction of withdrawers. We validate this view in a version of the standard Diamond and Dybvig [8] model, in which the fraction of impatient consumers is drawn stochastically according to a continuous density function, by showing that: (1) when banks are not allowed to suspend payments, in every symmetric equilibrium where agents deposit banks fail with strictly positive probability, and (2) in every such equilibrium, failure occurs whereas patient consumers find it optimal not to withdraw early. Moreover, we obtain similar results when banks are allowed to suspend payments, and we show that consumers' ex-ante welfare is strictly higher compared to when banks cannot suspend payments. Our contribution is therefore two-fold: (1) bank failures driven by large withdrawals can be explained by any fundamental shock that leads to an high fraction of withdrawers, and (2) suspension of payments might be a critical part of the protection of the banking system.

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“[T]he literature that started with Diamond and Dybvig [8] is unable to explain bank runs until now.” — James Peck, and Karl Shell [14].

1 Introduction

Throughout history, we observe many episodes of a large number of people withdrawing their deposits from a bank, forcing it to suspend withdrawals or even to fail. The standard interpretation, expressed in Diamond and Dybvig [8], is that those episodes constitute bank runs: all depositors believe that the banks will fail, making it optimal for all of them to withdraw as early as possible (i.e., to run to the bank).

An important aspect of a bank run is that depositors lose their confidence in the banking system: everybody panics and many go to the bank just because all the others are going. The common explanation expressed in the literature is that loss of consumers’ confidence is associated with sunspots. In this paper, we propose an alternative explanation of bank failures driven by large withdrawals: These will result from the lack of full back up of deposits in times when the number of depositors in need of funds is unusually high.¹ Furthermore, these bank runs can happen even when those who do not need funds will not panic and go to bank. Hence, we name this *non-panic* bank failures.

It is important to note that an unexpectedly large number of withdrawals will produce an outcome that is observationally equivalent to a bank run: a large number of people will appear in the bank, and the bank will fail or be forced to suspend payments. Thus, in practice, it will be difficult to distinguish between panic bank runs and non-panic bank failures. However, unlike bank runs, non-panic bank failures have a simple explanation — in a competitive banking system, banks will choose not fully back their deposits, and so, an high number of withdrawers will force the bank to fail. Thus, we suggest that bank failures driven by large withdrawals might have a fundamental explanation: any fundamental shock that leads to a large number of withdrawals, such as for instance large layoffs in the workforce, will ultimately

¹An alternative and conceptually equivalent view is that banks assets are not liquid enough to satisfy large demand for withdrawal. Such a situation occurs in the case of bad loans from banks for instance.

produce bank failures.²

Furthermore, as Diamond and Dybvig [8] also pointed out, banks can design deposit contracts capable of removing the incentives that depositors, not in need for funds, might have to run to the bank. A possible way banks have to prevent panic bank runs is to keep the right to suspend payments, a policy that many banks used in the past. Interestingly, in their analysis Diamond and Dybvig [8] showed that banks following a suspension of payments policy would not need to suspend withdrawals in equilibrium. Thus, it is puzzling to observe suspension of withdrawals in practice despite this theoretical recommendation.

The explanation that we give above can also be used to explain equilibrium suspension of payments: since banks choose not to fully back up their deposits, when the number of withdrawers happens to be unusually high, banks will be forced to suspend withdrawals.³ In other words, bank failures, and suspension of payments can occur as an equilibrium phenomenon independent of consumers' panic. Moreover, we show that the higher the interest rates offered by banks, the higher the likelihood of observing such phenomena in equilibrium. Therefore, as in Diamond and Dybvig [8], we observe a trade-off between liquidity offered by banks, and bank risk, a fundamental characteristic of the banking industry.

In more details, we consider an environment similar to the one in Diamond and Dybvig [8]. Consumers are separated in two distinct types: impatient consumers in need for liquid assets early, and patient consumers willing to postpone consumption further. We assume that the fraction of impatient consumers is random. In contrast with Diamond and Dybvig [8], we assume that every fraction of impatient consumers may occur with strictly positive probability, although an high fraction of impatient people is relatively unlikely.⁴

Consumers can deposit their initial endowment to a bank, or invest it in an assets market. Banks offer demand deposit contracts to depositors. The

²Wallace [17] has expressed a related idea in a similar model. See section 2 for a discussion.

³Note that this argument does not support Friedman's recommendation that banks should be required to fully back up their demand deposit contracts (see Friedman [9] for the details of this statement). In our framework, if banks choose to do so then all consumers would be strictly worse off from an ex-ante standpoint.

⁴In the Appendix we show that all of our results extend to the case in which there is an upper bound less than one on the fraction of impatient consumers.

banking system is assumed to be competitive; therefore in this case, as Peck and Shell [14] show, banks are left to maximize consumers' ex-ante welfare.

We then show that every symmetric equilibrium can be of two distinct types: either no consumer deposits, or every consumer deposits and patient consumers do not withdraw early. In every equilibrium such that all the consumers deposit, a bank failure occurs with strictly positive probability when the banks cannot suspend withdrawals.

A similar result hold when banks can suspend payments. There are two types of symmetric equilibria: one in which no consumer deposits, and another in which every consumer deposits and every patient consumer does not withdraw their deposits early. Still, in this last type of equilibria, banks suspend withdrawals with strictly positive probability.

In the above equilibria, suspension of payments, or bank failures, correspond to a situation where the fraction of impatient consumers is unexpectedly high. The main intuition behind this result is that banks find it optimal to offer a positive interest rate, and thus, to provide some risk sharing to consumers whenever an high fraction of impatient consumers is relatively unlikely, facing however a failure with relatively small probability (or suspension if allowed). In such case, a large number of consumers withdrawing their deposits force banks to suspend payments or even to fail.

This type of equilibria reflects some practical features of the banking system: banks offer liquidity (i.e., a positive interest rate) to depositors, people deposit, they withdraw only when they have to, and sometimes banks suspend withdrawals (or fail, if they cannot suspend withdrawals). Furthermore, an equilibrium of this type has the property that no consumer panics, and still suspension of payments (or bank failure) occurs with strictly positive probability. In this way, we depart from the traditional explanation of large withdrawals, bank failures and suspension of withdrawals. This, in turn, points to an additional fragility of the banking system, as banks can be forced to suspend withdrawals, or fail, even if the public's confidence is maintained.

However, we do not dismiss the possibility that equilibrium bank runs can occur. As Diamond and Dybvig [8] have shown, bank runs can occur due to sunspots; this is the case even when banks can suspend withdrawals, as shown by Peck and Shell [14]. In a model similar to Diamond and Dybvig [8], Adão and Temzelides [1] have shown that bank runs also occurs in some particular equilibria, independently of sunspots.

As discussed earlier, suspension of payments is a critical part of the pro-

tection of the banking system. For instance, this was a feature of the banking panic of 1907, but was not used during the Great Depression. However, at the beginning of the twentieth century, suspension of payments were viewed as undesirable and avoidable (see Wallace [17] for a detailed discussion of those historical facts). As importantly, we show that suspension of payments is also beneficial to consumers: we show that consumers' ex-ante welfare is strictly higher when banks are allowed to suspend payments. In fact, we show that symmetric equilibria can be ranked in the following manner: no-deposit equilibria are strictly Pareto-dominated by deposit equilibria without suspension of payments, and deposit equilibria without suspension of payments are strictly Pareto-dominated by deposit equilibria with suspension of payments. The intuition behind the Pareto superiority of suspension of payments is that it guarantees (a lower bound for the) consumption to patient consumers at the cost of reducing the probability that a impatient consumers will consume. Thus we support Wallace [17]'s recommendation that suspension of payments might be a critical part of the protection of every banking system.

2 Related Literature

Our model is very similar to that of Diamond and Dybvig [8]. Even though bank runs equilibria are not present in our model (every such equilibria are anticipated by consumers, preventing them from depositing), we establish the link between assets liquidity, interest rates and risk of bank failure or activation of suspension of payments.

Another similarity of our work with that of Diamond and Dybvig is on the type of contract we allow banks to offer to depositors.⁵ As in their work, we only study bank contracts that promise to pay the same to all early withdrawers until the bank fails, or until it fully suspends payments. Recently, Green and Lin in [10] and in [11] have criticized this *simple contracting* approach, by arguing that banks could design contracts that depositors would prefer to the simple demand deposits of Diamond and Dybvig. A similar approach is taken by Peck and Shell [14], which considered a class of contract that includes the simple demand deposits.

⁵We also assume the same trading restrictions (i.e., that depositors cannot sell their position at the bank) as in Diamond and Dybvig [8]. The importance of these trading restrictions have been analyzed by Jacklin [12].

The reason why we take the simple contracting approach of Diamond and Dybvig rather than the more general approach of Green and Lin, and of Peck and Shell is essentially the one put forth in Postlewaite and Vives [15]: since we observe demand deposits for so long, something in the environment must make these contracts optimal. Although we do not provide a formal analysis of what properties would lead to the optimality of deposit contracts in a Diamond and Dybvig environment, we point out that our message would remain if we allowed banks to offer at most *finitely* many consumption levels to early withdrawers. In this case, we would still obtain that banks failures occur with positive probability, and that full suspension of payments would be beneficial to consumers. Assuming that banks can only offer *finitely* many consumption levels to early withdrawers seems reasonable; any contract violating this assumption would be rather hard to manage in practice.⁶⁷

An important aspect of our paper is that we explain the above phenomena without referring to exogenous events, such as sunspots. Our view is that bank failure or suspension of payments are essentially driven by economic fundamentals, such as a surge in unemployment or poor investment performances from banks. One of the appeal of this approach is that explaining bank failures through sunspots carries the problem that sunspot equilibria are not robust to a more refined notion of equilibria, as Adão, and Temzelides [1] have shown.

Also, the idea that panics can occur without involving the whole population of consumers was first expressed in a specific example given in Postlewaite and Vives [15]. In this last reference, consumers find themselves in a

⁶Since Green and Lin, and Peck and Shell assume a finite number of consumers, this assumption will be automatically satisfied in their framework. In general, the optimal allocation in their framework will require as many consumption levels for early withdrawers as the number of consumers in the model. In a continuum of consumers model like our (and like Diamond and Dybvig's) the analog of their optimal contracts would require infinitely many consumption levels for early withdrawers. We take the view, as in Aumann [2], that the appropriate mathematical model to capture the intuitive notion of an economy with a very large number of participants, each of which having a negligible impact (as seems to be the case with the banking system in developed economies) is a model with a continuum of participants. In this case, the assumption that the bank can offer at most finitely many consumption levels to early withdrawers is just a technological assumption like the sequential service constraint: what is being assumed is that the technology of banks is such that an optimally organized bank will offer contract featuring these properties.

⁷Similar ideas are expressed in Carmona [4], in which monetary trading is rationalized as a Pareto efficient way to obtain certain properties including finite complexity.

prisoner' dilemma type of situation in terms of optimal withdrawing time, and individual decisions to run are based on anticipations of others' behaviors. Nevertheless, the link between macroeconomic fundamentals and bank failure is not established there, and the main explanation provided by the authors is that panics are triggered by highly sophisticated reasonings from consumers. Rather, our view is to emphasize that psychological factors have less explanatory power than macroeconomic fundamentals.

The main motivation of this paper is very similar to Wallace [17], where the author makes the following point: "In my model, the cause of a bank run and a partial suspension is exogenous — an aggregate shock to tastes that make the number of people wanting to withdraw unusually large." Still, this last reference relies on the assumption that there is a small amount of aggregate risk limited to a small group of individuals. Such a situation cannot reflect large-scale phenomena such as the Great Depression, and the explanatory power of this paper does not extend to what we consider to be the most natural causes of failure of the banking system, as described in Carmona and Leoni [6]. In contrast, our view allows to give a sensible explanation to large-scale phenomena missed in Wallace [17].⁸

Still, we do not dismiss the possibility that bank runs may be a consequence of herd behaviors, as in Chari and Jagannathan [7] and in Jacklin and Bhattacharya [13]. This view can also be found in Carmona and Leoni [5], where we show that when consumers observe the number of early withdrawers, then a large number of withdrawals will trigger panic, and in turn bank runs by a herd behavior effect. However, we show that such bank runs have a fundamental origin in economic fundamentals, a point that we want to emphasize. Some empirical support for this view can be found in Bernake [3]'s analysis of the US great depression.

The paper is organized as follows: in Section 3 we formally describe the model; in Section 4 we characterize symmetric equilibria and study their properties when banks are not allowed to suspend payments; in Section 5 we carry out the same analysis when banks are allowed to suspend payments

⁸From a methodological point of view, our work is closer to Wallace [16] since the model we use differs from the one used by Wallace only on the number of consumers (which is finite in the latter). A more substantial difference is on the restrictions we impose on the optimal contracting problem: we impose that the contract offered by the bank can offer at most finitely many consumption levels to early withdrawers, which will have consequences in our continuum of consumers model. Of course, this assumption does not place any restriction in Wallace's model.

and make explicit the welfare ranking of equilibria studied; finally in the Appendix we show that the assumption on the fraction of impatient consumers drawn can be relaxed in the manner described earlier.

3 The model

In this section, we formally describe the model. The model presented here is very similar to the one presented in Diamond and Dybvig [8].

The model has three periods ($T = 0, 1, 2$), and a single consumption good. There is a continuum of consumers; without loss of generality we assume that the set of consumers is represented by the interval $[0, 1]$. Every consumer is endowed with 1 unit of consumption good in period 0.

All consumers are identical in period 0. Every consumer can be of two distinct types, denoted by type 1 and type 2. A type 1 consumer values consumption in period 1 only (impatient consumer), whereas a type 2 consumer values consumption both in period 1 and 2 (patient consumer).

In period 1, nature draws a type for every consumer (also called an agent later). A fraction $t \in [0, 1]$ of the consumers will be of type 1. The random number t is drawn by nature according to a probability distribution f over $[0, 1]$. We assume that f is continuous, $f(1) = 0$, and $F(t) < 1$, for all $t \in [0, 1)$. Hence, every fraction of impatient consumers may occur, although an high fraction of impatient people is relatively unlikely. In period 0, the agents are equally likely to be of type 1 conditional on t , and the density of probability f is common knowledge among the agents. In period 1, every agent privately learns his type.

The agents can use their endowment in three different manners. Firstly, they can deposit in period 0 part of their endowment in a bank. With the deposit from the agents, the bank uses an investment technology exhibiting constant returns to scale. With this technology, one unit of consumption good invested in period 0 yields R ($R > 1$) units of consumption good in period 2. If the investment is withdrawn in period 1, the salvage value will be exactly the value of the investment.

Concurrently to the bank, there exists a storage technology described next. Every agent can privately store a quantity of consumption good of his choice in every period, in order to consume it next period. The storage technology is costless, and provides no return to the agents.

Finally, the agents have access to a competitive market for claims on

future goods, which is open in every period. In this market, it is shown in Diamond and Dybvig [8] that the period 0 price of period 1 consumption is 1, and the period 0 and 1 prices of period 2 consumption are R^{-1} . We assume that, when the agents are indifferent between depositing at the bank and investing in the assets market, the agent will always prefer to invest in the assets market.

Let c_1 denote individual consumption of an agent in period 1, let c_2 denote individual consumption in period 2, and let Θ be the type of the agent. The utility derived by every agent from the consumption of the bundle (c_1, c_2) is

$$U(c_1, c_2, \Theta) = \begin{cases} u(c_1) & \text{if the agent is of type 1} \\ u(c_1 + c_2) & \text{otherwise} \end{cases} \quad (1)$$

where $u : \mathbb{R}_+ \rightarrow \mathbb{R}$ is twice continuously differentiable, strictly increasing, strictly concave, and satisfies the Inada conditions $u'(0) = \infty$, and $u'(\infty) = 0$. Also, we assume the following on the relative risk-aversion coefficient: $-cu''(c)/u'(c) > 1$ for $c \geq 1$. We also normalize units so that $u(0) = 0$. Every agent is assumed to maximize the ex-ante (relative to period 0) expected utility $E[U(c_1, c_2, \Theta)]$.

We next describe the banking industry in more details. There is a large number of banks, behaving competitively (see Shell and Peck [14] for the implications of such an assumption). The banks offer *demand deposit contracts* to their depositors; that is, the banks offer to the depositors a contract specifying a fixed claim of r_1 per unit deposited to agents withdrawing in period 1. The banks are mutually owned, and they are liquidated in period 2. This implies that period 2 withdrawer will equally share among themselves the remainder of the banks assets.

The banks also satisfy a *sequential service constraint*; that is, the banks must serve the agents withdrawing in period 1 in the order that they arrive (randomly) at the bank until the bank runs out of assets.

Let A denote the total amount of deposit in period 0, and consider an agent $j \in [0, 1]$ willing to withdraw in period 1. Let f_j denote the number of period 1 withdrawer arriving at the bank before agent j , and let V_1 denote the period 1 payoff per unit deposit to this agent j . We have that

$$V_1(f_j, r_1) = \begin{cases} r_1 & \text{if } f_j < r_1^{-1}A \\ 0 & \text{otherwise.} \end{cases} \quad (2)$$

Let now V_2 denote the period 2 payoff per unit deposit not withdrawn in period 1, and let f be the number of demand deposits withdrawn in period

in period 1. We have that

$$V_2(f, r_1) = \max \left[\frac{R(A - r_1 f)}{1 - f}, 0 \right]. \quad (3)$$

Denoting by w_j the fraction of deposit withdrawn in period 1 by agent j (for every $j \in [0, 1]$), and assuming that agent j deposits all of its endowment in the bank, the overall payoff to agent j is then given by

$$w_j V_1(f_j, r_1) + (1 - w_j) V_2(f, r_1). \quad (4)$$

A strategy of the banks is the choice of $r \in [1, R]$; the banks choose r in order to maximize ex-ante utility of the consumers (recall that they are equal ex-ante). This behavior is motivated by the competitive nature of the banking industry (See Shell and Peck [14] for more details on this issue). More generally, we could assume that the banks offer $r \in \mathbb{R}$, but risk-aversion will ensure that $r \in [1, R]$.

Consumers will choose individually whether to deposit (if they don't, they will buy claims for future goods), for all possible interest rates the bank offer. Also, for every possible interest rate, and deposit choice, a consumer will choose either to withdraw in the first period or to wait, conditional on his type and others' strategies. Hence, a strategy for a consumer is d, w , where d is a function from $[1, r]$ into $\{0, 1\}$, and w is a function from $[1, R] \times \{0, 1\} \times \Theta$ into $\{0, 1\}$. We make the convention that $d(r) = 1$ stands for the choice of depositing, and similarly $w(r, d, \Theta) = 1$ means that she will withdraw in period 1.

In order to evaluate different strategies, each consumer needs to know the probability of arriving to the bank before this fails. Conditional on the fraction t of impatient consumers in the population, this probability depends on the interest rate r offered by the bank, on the fraction s at which the bank suspends withdraws (if allowed), and on the strategies chosen by the other consumers. Since we are focusing only on symmetric strategies, it is enough for a consumer to know this probability in the three following cases: 1) when all consumers withdraw in period 1, 2) when only impatient consumers withdraw in period 1, and 3) when he withdraws in period 1 in addition to all impatient consumers — these functions will be denoted respectively by $\alpha_a(r, s|t)$, $\alpha_i(r, s|t)$, and $\alpha_1(r, s|t)$. We assume that for all $y = a, i, 1$, the function α_y is continuous with respect to t , is differentiable with respect to

s with bounded partial derivative, and that

$$\int_0^1 \alpha_a(r, s|t) r f(t) dt \leq 1.^9 \quad (5)$$

An *equilibrium* is then r^* , d^* , and w^* such that $w^*(r, d, \Theta)$ is optimal for all (r, d, Θ) , $d^*(r)$ is optimal for all r , and r^* is optimal taking as given agents' strategies.

A *bank failure* occurs when the total value of assets withdrawn in period 1 strictly exceeds the total value of assets owned by the bank.

4 Bank Failures

In this section, we characterize the symmetric equilibria where banks are not allowed to suspend withdrawals, and we study their properties.

We first show that such equilibria can be of two distinct types: either agents avoid depositing, or deposit and patient agents wait. We also show that agents' welfare is strictly higher in the second class of equilibria.

Finally, we show that, in every equilibrium such that every agent deposits, a bank failure occur with strictly positive probability. Moreover, we show that this probability is a function of equilibrium interest rates offered by banks.

For sake of simplicity, we carry out the analysis with the assumption that every fraction of impatient agents can be drawn with strictly positive probability. In the Appendix, we show that this assumption can be relaxed in order to get the following result.

Proposition 1 *There are two classes of symmetric equilibria:*

1. (*autarkic equilibria*) no agent deposits; i.e., $d^*(r^*) = 0$.
2. (*non-autarkic equilibria*) every agent deposits, all patient agents wait, and the interest offered by the bank strictly exceeds 1; i.e., $r^* > 1$, $d^*(r^*) = 1$, and $w^*(r^*, d^*(r^*), 2) = 0$.

⁹It will become clear later that the assumptions can be relaxed substantially. See the appendix for an explicit probabilistic model yielding functions α_y , $y = a, i, 1$ for which all our results hold.

In every non-autarkic equilibrium, the agents' ex-ante expected utility is strictly greater than in any autarkic equilibrium. Furthermore, in every non-autarkic equilibrium, the banks fail in the first period with strictly positive probability (given by $1 - F(1/r^)$).¹⁰*

In particular, this proposition shows that in every non-autarkic equilibria, bank runs can occur even when no patient consumers wishes to withdraw early. Furthermore, despite the risk of a bank failure, everyone finds optimal to deposit their endowment in the bank. Because of this two properties, we label this type of equilibria as non-panic bank failures.

Proceeding with the proof of Proposition 1, we note first that all impatient consumers will choose to withdraw in period 1. Thus, in a symmetric equilibrium, we have only four possibilities regarding consumer's behavior:

1. no consumer deposits, and patient consumers withdraw in period 1,
2. no consumer deposits, and patient consumers withdraw in period 2,
3. all consumers deposit, and patient consumers withdraw in period 1, and,
4. all consumers deposit, and patient consumers withdraw in period 2.

Clearly, cases 1, and 2 above are an equilibrium, for any interest rate offered by the bank — these two cases correspond to part 1 of Proposition 1.

It is also clear that if every consumer deposits, then it cannot be the case that both patient and impatient consumers will withdraw — i.e., that case 3 above is not possible in equilibrium. This follows because any consumer would prefer not to deposit given this strategy as $\int_0^1 \alpha_a(r, 0|t)rf(t)dt \leq 1$, and consumers are risk averse. Thus, in order to complete the proof of Proposition 1, we are left to show that there exist an equilibrium in which patient consumers wait, and that all of these equilibria have the stated properties.

We first construct an equilibrium in which all consumers deposit, and all patient consumers wait to withdraw in period 2. Unlike the three previous cases, in this case consumers behavior depends on the interest rate offered by the bank. In particular, suppose that the bank offer an interest rate equal

¹⁰A more precise statement is as follows: if S denotes the set of symmetric equilibria, A the set of symmetric equilibria described in 1, and B the set of symmetric equilibria described in 2, we have that $S = A \cup B$, $A \cap B = \emptyset$, $A \neq \emptyset$, and $B \neq \emptyset$.

to r , all consumers deposit, and all patient consumers wait for the second period to withdraw. Then, the expected utility for a patient consumers equal

$$\int_0^1 u \left(\max \left\{ \frac{R(1-tr)}{1-t}, 0 \right\} \right) f(t) dt. \quad (6)$$

If one patient consumer decides to withdraw in period 1, then his expected utility when all the other patient consumer withdraw in period 2 is given by

$$\int_0^1 u(r) \alpha_1(r|t) f(t) dt. \quad (7)$$

Thus, letting

$$W = \left\{ r \in [1, R] : \int_0^1 u \left(\max \left\{ \frac{R(1-tr)}{1-t}, 0 \right\} \right) f(t) dt \geq \int_0^1 u(r) \alpha_1(r|t) f(t) dt \right\}, \quad (8)$$

we have that any patient consumer will choose to withdraw in period 2 provided that r belongs to W , all consumers deposits, and all other patient consumers wait for period 2 to withdraw.

From the above, we conclude that in an equilibrium of the type described in case 4, a patient consumers receives an utility of

$$u \left(\max \left\{ \frac{R(1-tr)}{1-t}, 0 \right\} \right), \quad (9)$$

and a impatient consumer receives an utility of

$$\alpha_i(r|t) u(r), \quad (10)$$

when the fraction of impatient consumers is equal to t . Thus, the ex-ante expected utility of any consumer in this type of equilibrium is

$$U(r) := \int_0^1 \left[t \alpha_i(r|t) u(r) + (1-t) u \left(\max \left\{ \frac{R(1-tr)}{1-t}, 0 \right\} \right) \right] f(t) dt. \quad (11)$$

If a consumer decides not to deposit, then his ex-ante expected utility is simply

$$\int_0^1 [t u(1) + (1-t) u(R)] f(t) dt. \quad (12)$$

Hence, letting

$$D = \left\{ r \in [1, R] : U(r) \geq \int_0^1 [tu(1) + (1-t)u(R)] f(t)dt \right\}, \quad (13)$$

we see that any consumer will choose to deposit provided that r belongs to D , all other consumers deposits, and all patient consumers wait for period 2 to withdraw.

Finally, if the bank offers r^* which maximizes U in the set $D \cap W$, we can construct a symmetric equilibrium in the following way: the bank offers r^* , and the consumers choose

$$d^*(r) = \begin{cases} 1 & \text{if } r \in D \cap W \\ 0 & \text{otherwise,} \end{cases} \quad (14)$$

$$w^*(r, d, 2) = \begin{cases} 0 & \text{if } r \in D \cap W \text{ and } d = 1 \\ 1 & \text{otherwise,} \end{cases} \quad (15)$$

and $w^*(r, d, 1) = 1$ for all (r, d) .

Of course, in order to be able to construct the above strategy, we need to show the existence of r^* . This is done in the following lemma.

Lemma 1 *The function U has a maximizer in $D \cap W$.*

Proof. Note that the set $D \cap W$ is compact, and non-empty, since $r = 1$ belong to $D \cap W$. The function U is a continuous function of r . Hence, there exist r^* that maximizes U in $D \cap W$. ■

We have shown so far that there are two types of equilibria: in one type nobody deposits, and in the other type everyone deposits, and patient consumers withdraw in period 2. In order to complete the proof of Proposition 1, we are left to show that in every equilibrium of the second type, we have $r^* > 1$. Note that $r^* > 1$ implies a strictly positive probability of a bank failure: since the bank has to pay r^*t in the first period, we see that, with probability $1 - F(1/r^*) > 0$, the banks fails.

Lemma 2 *In every symmetric equilibrium such that every agent deposits, and all patient agents wait, we have $r^* > 1$.*

Proof. It suffices to show that there exists $\tilde{r} > 1$ such that \tilde{r} belongs to $D \cap W$, and $U(\tilde{r}) > U(1)$, which in turn implies that $r^* > 1$.

Let $WL(r)$ (resp. $WR(r)$) denote the left-hand (resp. right-hand) side of the inequality defining the set W . Since $WL(1) = u(R) > u(1) = WR(1)$, we conclude that there exists a ball B around of 1 contained in W .

Note that $D = \{r \in [1, R] : U(r) \geq U(1)\}$. Therefore, to prove existence of $\tilde{r} > 1$ such that $\tilde{r} \in D \cap W$, and $U(\tilde{r}) > U(1)$, it is enough to show that

$$\lim_{r \searrow 1} \frac{U(r) - U(1)}{r - 1} > 0. \quad (16)$$

This is so, because if equation 16 holds, then it cannot be the case that $\frac{U(r) - U(1)}{r - 1} \leq 0$ for all $r > 1$ in the ball B around 1. Thus, there exist $\tilde{r} > 1$ in $B \subseteq W$ such that $\frac{U(\tilde{r}) - U(1)}{r - 1} > 0$; this, of course, implies that $U(\tilde{r}) > U(1)$, and $\tilde{r} \in D$.

Let $g(r) = R(1 - tr)/(1 - t)$. We have that

$$\begin{aligned} \frac{U(r) - U(1)}{r - 1} = & \int_0^{1/r} \left[t \frac{u(r) - u(1)}{r - 1} + (1 - t) \frac{u \circ g(r) - u \circ g(1)}{r - 1} \right] f(t) dt + \\ & \frac{1}{r - 1} \int_{1/r}^1 [t(\alpha_i(r|t)u(r) - u(1)) - (1 - t)u(R)] f(t) dt. \end{aligned} \quad (17)$$

Since the following holds

$$\begin{aligned} & \frac{1}{r - 1} \int_{1/r}^1 [t(\alpha_i(r|t)u(r) - u(1)) - (1 - t)u(R)] f(t) dt \geq \\ & \frac{1}{r - 1} \int_{1/r}^1 [t(-u(1)) + (1 - t)(-u(R))] f(t) dt \geq \\ & -u(R) \frac{1}{r - 1} \int_{1/r}^1 f(t) dt = \\ & -u(R) \frac{F(1) - F(1/r)}{r - 1} \rightarrow -f(1)u(R) = 0, \end{aligned} \quad (18)$$

we are left to show that

$$\lim_{r \searrow 1} \int_0^{1/r} \left[t \frac{u(r) - u(1)}{r - 1} + (1 - t) \frac{u \circ g(r) - u \circ g(1)}{r - 1} \right] f(t) dt > 0. \quad (19)$$

Note that $g'(r)u'(g(r)) = -u'(R)Rt/(1-t)$. Defining

$$h_r(t) = \begin{cases} \left[t \frac{u(r)-u(1)}{r-1} + (1-t) \frac{u \circ g(r) - u \circ g(1)}{r-1} \right] f(t) & \text{if } t \in [0, 1/r] \\ 0 & \text{otherwise,} \end{cases} \quad (20)$$

we see that $\lim_{r \rightarrow 1} h_r(t) = [u'(1) - Ru'(R)]tf(t)$. Thus, by the Lebesgue Dominated Convergence Theorem, we obtain

$$\begin{aligned} \lim_{r \searrow 1} \int_0^{1/r} \left[t \frac{u(r)-u(1)}{r-1} + (1-t) \frac{u \circ g(r) - u \circ g(1)}{r-1} \right] f(t) dt = \\ \lim_{r \searrow 1} \int_0^1 h_r(t) dt = \\ [u'(1) - Ru'(R)] \int_0^1 tf(t) dt > 0, \end{aligned} \quad (21)$$

since $u'(1) > Ru'(R)$ (see Diamond and Dybvig [8], footnote 2) and $\int_0^1 tf(t) dt > 0$. ■

Proposition 1 fully analyzes symmetric equilibria where banks are not allowed to suspend payments. We next study the case where suspension is allowed.

5 Suspension of Withdrawals

We now consider the case where banks are allowed to suspend withdrawals, and where every agent deposits. The banks are free to suspend withdrawals when their assets reach a fraction $s \in [0, 1]$ of the deposits. (The case where the banks choose not to suspend withdrawals corresponds to $s = 0$.) The choice of the fraction s is part of the banks' strategy. An *equilibrium* in this new framework is similar to the previous one, with the additional property that the fraction s chosen by the banks is now an equilibrium variable.

It is straightforward to see that, when banks are allowed to suspend withdrawals, bank failures are ruled out in any equilibrium where all the agents deposit.

We next carry out a welfare comparison of the systems with and without suspension of withdrawals. The next proposition shows that under a system with suspension of withdrawals, agents' welfare is strictly higher than under a system without suspension of withdrawals. Together with the previous

results, this last proposition strongly suggests that suspension of withdrawals might be a critical part of the protection of the banking system.

In order to prove that agents' welfare is strictly higher when banks are allowed to suspend withdrawals, we proceed as follows. Given that banks maximize ex-ante welfare of the agents, and given that not suspending withdrawals is part of the banks' strategies ($s = 0$), we show that in every equilibrium banks will choose suspension. Moreover, we show that agents' welfare is strictly increasing in a positive neighborhood of $s = 0$.

Let $U(r, s)$ denote consumers' ex-ante expected utility when the bank offer an interest rate r , and suspends payments at s .

Proposition 2 *Let $r > 1$ be such that every agent chooses to deposit ($d^*(r, 0) = 1$), and such that every patient agent chooses not to withdraw in period 1 ($w^*(r, 0, d^*(r, 0), 2) = 0$). Then there exists $s^* > 0$ such that $d^*(r, s^*) = 1$, $w^*(r, s^*, d^*(r, s^*), 2) = 0$ and*

$$U(r^*, s^*) > U(r^*, 0). \quad (22)$$

Proof. Note first that by suspending withdrawals, banks give more incentives for a patient consumer to wait. Thus, we are left to show that the bank can increase consumers' ex-ante utility by suspending withdrawals.

We have that

$$U(r, 0) = \int_0^{\frac{1}{r}} \left[tu(r) + (1-t)u\left(\frac{R(1-tr)}{1-t}\right) \right] f(t)dt + \int_{\frac{1}{r}}^1 t\alpha_i(r, 0|t)u(r)f(t)dt, \quad (23)$$

and that

$$U(r, s) = \int_0^{\frac{1-s}{r}} \left[tu(r) + (1-t)u\left(\frac{R(1-tr)}{1-t}\right) \right] f(t)dt + \int_{\frac{1-s}{r}}^1 \left[t\alpha_i(r, s|t)u(r) + (1-t)u\left(\frac{R(1-t'r)}{1-t'}\right) \right] f(t)dt, \quad (24)$$

where $t' = (1-s)/r$.

Then we obtain that

$$\begin{aligned}
U(r, s) - U(r, 0) &= \int_{\frac{1-s}{r}}^{\frac{1}{r}} [t(\alpha_i(r, s|t) - 1)u(r)] f(t)dt \\
&+ \int_{\frac{1-s}{r}}^{\frac{1}{r}} \left[(1-t) \left(u \left(\frac{R(1-t'r)}{1-t'} \right) - u \left(\frac{R(1-tr)}{1-t} \right) \right) \right] f(t)dt \\
&+ \int_{\frac{1}{r}}^1 \left[t(\alpha_i(r, s|t) - \alpha_i(r, 0|t))u(r) + (1-t)u \left(\frac{R(1-t'r)}{1-t'} \right) \right] f(t)dt.
\end{aligned} \tag{25}$$

We claim that

$$\lim_{s \rightarrow 0^+} \frac{U(r, s) - U(r, 0)}{s} = +\infty, \tag{26}$$

which implies the existence of $s^* > 0$ such that $U(r^*, s^*) > U(0)$. This in turn implies that $s^* > 0$.

Note first that

$$\frac{1}{s} \int_{\frac{1}{r}}^1 (1-t)u \left(\frac{R(1-t'r)}{1-t'} \right) f(t)dt = \int_{\frac{1}{r}}^1 (1-t) \frac{u \left(\frac{R(1-t'r)}{1-t'} \right) - u(0)}{s} f(t)dt. \tag{27}$$

Since $u'(0) = +\infty$, and $\lim_{s \rightarrow 0^+} R(1-t'r)/(1-t') = 0$ we see that this term converges to infinity as s converges to 0.

Furthermore,

$$\begin{aligned}
\lim_{s \rightarrow 0^+} \frac{1}{s} \int_{\frac{1}{r}}^1 t(\alpha_i(r, s|t) - \alpha_i(r, 0|t))u(r)f(t)dt = \\
\int_0^1 t \frac{\partial \alpha_i(r, 0|t)}{\partial s} u(r)f(t)dt,
\end{aligned} \tag{28}$$

and so, we conclude that this term is bounded below. Hence, to finish the proof it is enough to show that

$$\begin{aligned}
&\frac{1}{s} \int_{\frac{1-s}{r}}^{\frac{1}{r}} [t(\alpha_i(r, s|t) - 1)u(r)] f(t)dt \\
&+ \frac{1}{s} \int_{\frac{1-s}{r}}^{\frac{1}{r}} \left[(1-t) \left(u \left(\frac{R(1-t'r)}{1-t'} \right) - u \left(\frac{R(1-tr)}{1-t} \right) \right) \right] f(t)dt
\end{aligned} \tag{29}$$

is bounded below.

The continuity of α_i and u allow us to find $B \in \mathbb{R}$ such that

$$t(\alpha_i(r, s|t) - 1)u(r) + (1-t)\left(u\left(\frac{R(1-t'r)}{1-t'}\right) - u\left(\frac{R(1-tr)}{1-t}\right)\right) \geq B, \quad (30)$$

for all $t \in [0, 1]$. Thus, it follows that

$$\begin{aligned} & \frac{1}{s} \int_{\frac{1-s}{r}}^{\frac{1}{r}} [t(\alpha_i(r, s|t) - 1)u(r)] f(t) dt \\ & + \frac{1}{s} \int_{\frac{1-s}{r}}^{\frac{1}{r}} \left[(1-t) \left(u\left(\frac{R(1-t'r)}{1-t'}\right) - u\left(\frac{R(1-tr)}{1-t}\right) \right) \right] f(t) dt \quad (31) \\ & \geq \frac{B}{s} \int_{\frac{1-s}{r}}^{\frac{1}{r}} f(t) dt = B \frac{F(\frac{1}{r}) - F(\frac{1-s}{r})}{s} \rightarrow \frac{B}{r} f\left(\frac{1}{r}\right). \end{aligned}$$

This completes the proof. ■

The above result states that in every equilibrium such that agents deposit, banks will chose to suspend payments. We are left to show that such an equilibrium exists. An important feature of proposition 1 is existence of such an equilibrium when suspension is not allowed.

We next establish a similar result when suspension is allowed. Still, in every such equilibrium, it is straightforward to see that suspension will occur with strictly positive probability.

However, in the Appendix, we show that the next result obtain with a weakened assumption of the density of probability f (namely, the next result holds when a fraction of patient agents with strictly positive measure is drawn with probability 1).

The intuition and motivation for this next result are similar to the ones in proposition 1.

Proposition 3 *There exists an equilibrium such that $r^{**} > 1$, $s^{**} > 0$, $d^*(r^*) = 1$, and $w^*(r^*, d^*(r^*), 2) = 0$.*

*The consumers' ex-ante welfare with (r^{**}, s^{**}) is strictly higher then with $(r^*, 0)$, where r^* is as in Proposition 1. Furthermore, the probability that the bank suspend payments in this equilibrium is strictly positive.*

Proof. Let

$$\mathcal{W} = \left\{ (r, s) \in [1, R] \times [0, 1] : \int_0^{\frac{1-s}{r}} u \left(\frac{R(1-tr)}{1-t} \right) f(t) dt + \left(1 - F \left(\frac{1-s}{r} \right) \right) u \left(\frac{R(1-s)}{1-\frac{1-s}{r}} \right) \geq \int_0^1 u(r) \alpha_1(r, s|t) f(t) dt \right\}, \quad (32)$$

and let

$$\mathcal{D} = \left\{ (r, s) \in [1, R] \times [0, 1] : U(r, s) \geq \int_0^1 [tu(1) + (1-t)u(R)] f(t) dt \right\}, \quad (33)$$

where

$$U(r, s) := \int_0^{\frac{1-s}{r}} \left[tu(r) + (1-t)u \left(\frac{R(1-tr)}{1-t} \right) \right] f(t) dt + \int_{\frac{1-s}{r}}^1 \left[t\alpha_i(r, s|t)u(t) + (1-t)u \left(\frac{R(1-t'r)}{1-t'} \right) \right] f(t) dt, \quad (34)$$

and $t' = (1-s)/r$.

We have that, for every $(r, s) \in \mathcal{D} \cap \mathcal{W}$, every consumer deposits, and every patient consumer does not withdraw early.

Consider now the following strategy: the bank offers (r^{**}, s^{**}) that maximizes $U(r, s)$ in $\mathcal{D} \cap \mathcal{W}$, and consumers choose

$$d^*(r, s) = \begin{cases} 1 & \text{if } (r, s) \in \mathcal{D} \cap \mathcal{W} \\ 0 & \text{otherwise,} \end{cases}$$

$$w^*(r, s, d, 2) = \begin{cases} 0 & \text{if } (r, s) \in \mathcal{D} \cap \mathcal{W} \text{ and } d = 1 \\ 1 & \text{otherwise,} \end{cases}$$

and $w^*(r, s, d, 1) = 1$ for all (r, s, d) .

To prove existence of an equilibrium with the above properties, it is enough to show that the following program

$$\begin{aligned} & \max_{r, s} U(r, s) \\ & \text{subject to } (r, d) \in \mathcal{D} \cap \mathcal{W} \end{aligned}$$

has a solution.

To do so, it is straightforward to show that both sets \mathcal{W} and \mathcal{D} are compact. Moreover, $(r, s) = (1, 0)$ belong to $\mathcal{D} \cap \mathcal{W}$. Therefore, the set $\mathcal{D} \cap \mathcal{W}$ is nonempty and compact. Since the objective function is continuous on this last set, the program has a solution.

Let r^* be as in Proposition 1. Then, $(r^*, 0)$ belong to $\mathcal{D} \cap \mathcal{W}$, and, for any $s > 0$, we have

$$U(1, s) = U(1, 0) < U(r^*, 0).$$

Hence, the solution to the above problem (r^{**}, s^{**}) , will have $r^{**} > 1$, and therefore, $s^{**} > 0$, by Proposition 2. The proof is now complete. ■

6 Conclusion

We used the standard Diamond and Dibvig [8] to show that bank runs (associated with the idea of consumers' panic) are not the only theoretical explanation for bank failures driven by large withdrawals. We show that even when consumers do not panic, an unexpectedly high level of consumers in need for liquid assets early may lead the whole banking industry to fail. Moreover, allowing for suspension of payments achieves strictly higher consumers' welfare. It follows that, at least from a theoretical standpoint, suspension of payments might be a critical part of the protection of the banking system.

The main point of this paper is that bank failures, and suspension of payments, can have a fundamental explanation. Any (relatively unlikely) fundamental shock that leads to a large number of withdrawals, will lead to bank failures, or to suspension of payments. Hence, we suggest that future research should focus on what fundamental shocks can lead to large withdrawals. This is a challenge we take on Carmona and Leoni [6], where we suggest that such a fundamental shock might be a severe recession.

A Appendix

A.1 Alpha

In this appendix, we present a model yielding functions α_y , $y = a, i, 1$ for which all our results hold. Let nature first chooses t according to f . Then, independently, it chooses $t_0 \in [0, 1]$ according to the uniform distribution, and picks consumers in $[t_0, (t_0 + t) \bmod 1]$ to be impatient (it is convenient to think of the interval $[0, 1]$ as a circle). Independently of t_0 , and of t , nature chooses $p \in [0, 1]$, again according to the uniform distribution, to be the first consumer to arrive at the bank, which will be followed by those to his right (modulo 1).¹¹

By symmetry, it is enough to consider consumer 0. If all consumer withdraw in the first period, then consumer 0 will be able to consume provided that $1 - (1 - p)r \geq s$, that is, $p \geq 1 - (1 - s)/r$. Thus,

$$\alpha_a(r, s|t) = \frac{1 - s}{r}. \quad (36)$$

For the case when only impatient consumers withdraw in period 1, assuming that consumer 0 is impatient implies that $t_0 \in [1 - t, 1]$; thus, conditioning will be needed. Assume first that $(1 - s)/rt \leq 1$. We obtain that if $t_0 \in [1 - t, 1 - (1 - s)/r]$, then for consumer 0 to be able to withdraw we need $1 - (1 - p)r \geq s$, that is, $p \geq 1 - (1 - s)/r$. If $t_0 \in [1 - (1 - s)/r, 1]$ then p need to be greater or equal to x , where x solves

$$1 - r[(1 - t_0) + (t_0 + t - 1) - x] = s \quad (37)$$

, and so $p \geq t - (1 - s)/r$. Hence,

$$\begin{aligned} \alpha_i(r, s) &= \frac{1}{t} \left(\int_{1-t}^{1-\frac{1-s}{r}} \frac{1-s}{r} dt_0 + \int_{1-\frac{1-s}{r}}^1 \left(1 - t + \frac{1-s}{r} \right) dt_0 \right) \\ &= \frac{1-s}{rt}. \end{aligned} \quad (38)$$

¹¹More precisely, the purpose of the variable p is to define an order on $[0, 1]$, as follows: consumer x arrives to the bank before consumer z if $h(x) < h(z)$, where

$$h(x) = \begin{cases} x - 0.5 & \text{if } x \geq p \\ x + 0.5 & \text{otherwise.} \end{cases} \quad (35)$$

Finally, if $(1 - s)/rt > 1$, then clearly, $\alpha_i(r, s|t) = 1$. Thus,

$$\alpha_i(r, s|t) = \min \left\{ \frac{1 - s}{tr}, 1 \right\}. \quad (39)$$

Finally, we have the case when one patient consumer withdraw in period 1 in addition to all the impatient ones. We obtain

$$\alpha_1(r, s) = \begin{cases} \frac{1-t}{2} + \frac{1-s}{r} & \text{if } rt \leq 1 - s \\ 1 & \text{otherwise.} \end{cases} \quad (40)$$

The last expression can be obtained as follow: again suppose that consumer 0 is the patient consumer who chooses to withdraw in period 1. This implies that t_0 can only be between 0, and $1 - t$. Given t_0 , then he can withdraw if and only if $1 - r(t_0 + t - p) \geq s$, that is, if and only if, $p \geq t_0 + t - (1 - s)/r$. Thus,

$$\alpha_1(r, s|t) = \frac{1}{1 - t} \int_0^{1-t} (1 - t_0 - t + \frac{1 - s}{r}) dt_0 = \frac{1 - t}{2} + \frac{1 - s}{r}. \quad (41)$$

One can easily show that all these function are continuous almost everywhere (with respect to the Lebesgue measure) separately on (r, s) , and t . Furthermore, the partial derivative of α_i with respect to s exists and is bounded almost everywhere. As one can also easily check, these properties are sufficient for our results.

A.2 Robustness

In this section, we show that the strictly positive probability of an equilibrium bank failure in proposition 1 does not depend on the assumption that every fraction of impatient consumers has strictly positive probability. We now assume that there exists a maximal fraction (strictly less than one) of impatient agents possibly drawn, and we show that a result identical to proposition 1 obtains.

We can also obtain similar results to Proposition 1 and Proposition 2 with this last assumption on the distribution of patient agents. For sake of brevity, we omit them.

Before presenting the next result, we introduce the following notation: Let $t \in [0, 1]$, and let $(c_1^*(t), c_2^*(t))$ be the solution to

$$\begin{aligned} & \max_{c_1(t), c_2(t)} tu(c_1(t)) + (1-t)u(c_2(t)) \\ & \text{subject to } tc_1(t) + \frac{(1-t)c_2(t)}{R} = 1. \end{aligned} \quad (42)$$

We next state the result.

Proposition 4 *Suppose that there exists $\bar{t} \in (0, 1)$ such that $F(\bar{t}) = 1$, f is continuous, $F(t) < 1$ for all $t \in [0, \bar{t})$, and $f(\bar{t}) = 0$. Let $\bar{r} = c_1^*(\bar{t})$.*

There are two classes of symmetric equilibria:

1. *no agent deposits; i.e., $d^*(r^*) = 0$.*
2. *every agent deposits, every patient agent waits, and the interest rate offered by the banks exceed 1; i.e., $r^* > \bar{r}$, $d^*(r^*) = 1$, and $w^*(r^*, d^*(r^*), 2) = 0$. In every equilibrium in this class, the ex-ante expected utility of the agents is strictly greater than in the previous class. Moreover, the banks fail in the first period with strictly positive probability (equal to $1 - F(1/r^*)$).*

Proof. The proof is almost similar to the proof of proposition 1. We only need show that: (a) $U(r) < U(\bar{r})$ if $r < \bar{r}$, and (b) there exists $\tilde{r} > \bar{r}$ such that \tilde{r} belongs to $D \cap W$, and $U(\tilde{r}) > U(\bar{r})$.

In order to prove (a), we next show that

$$tu(r) + (1-t)u\left(\frac{R(1-tr)}{1-t}\right) < tu(\bar{r}) + (1-t)u\left(\frac{R(1-t\bar{r})}{1-t}\right), \quad (43)$$

for every $r < \bar{r}$, and every $t < \bar{t}$.

Pick $r < \bar{r}$, and also pick $t < \bar{t}$. Consider the following problem:

$$\begin{aligned} & \max_{c_1, c_2} tu(c_1) + (1-t)u(c_2) \\ & \text{subject to } tc_1 + \frac{(1-t)c_2}{R} = 1 \\ & \text{and } c_1 \leq \bar{r}. \end{aligned} \quad (44)$$

A necessary condition for a maximum is that

$$[c_1 - \bar{r}] \left[u'(c_1) - Ru' \left(\frac{R(1-t\bar{r})}{1-t} \right) \right] = 0. \quad (45)$$

Since $c_1 \leq \bar{r}$, and $t < \bar{t}$, it follows that

$$u'(c_1) \geq u'(\bar{r}) = Ru' \left(\frac{R(1 - \bar{t}\bar{r})}{1 - \bar{t}} \right) > Ru' \left(\frac{R(1 - tc_1)}{1 - t} \right), \quad (46)$$

and so $c_1 = \bar{r}$. Thus,

$$tu(r) + (1 - t)u \left(\frac{R(1 - tr)}{1 - t} \right) < tu(\bar{r}) + (1 - t)u \left(\frac{R(1 - t\bar{r})}{1 - t} \right). \quad (47)$$

In order to prove (b), we note first that $\bar{r} \in D \cap W$. By (1), we see that $U(\bar{r}) > U(1)$, and so D contains a neighborhood of \bar{r} . Similarly, W contains a neighborhood of \bar{r} , since

$$\begin{aligned} LW(\bar{r}) &= \int_0^{\bar{t}} u \left(\frac{R(1 - t\bar{r})}{1 - t} \right) f(t) dt \\ &> \int_0^{\bar{t}} u(\bar{r}) f(t) dt \\ &= RW(\bar{r}). \end{aligned} \quad (48)$$

Finally, we will show that

$$\lim_{r \searrow \bar{r}} \frac{U(r) - U(\bar{r})}{r - \bar{r}} > 0, \quad (49)$$

which implies the existence of $\tilde{r} > \bar{r}$ such that $\tilde{r} \in D \cap W$, and $U(\tilde{r}) > U(\bar{r})$.

Let $g(r) = R(1 - tr)/(1 - t)$. We have that

$$\begin{aligned} \frac{U(r) - U(\bar{r})}{r - \bar{r}} &= \\ &\int_0^{1/r} \left[t \frac{u(r) - u(\bar{r})}{r - \bar{r}} + (1 - t) \frac{u \circ g(r) - u \circ g(\bar{r})}{r - \bar{r}} \right] f(t) dt + \\ &\frac{1}{r - \bar{r}} \int_{1/r}^{\bar{t}} [t(\alpha_i(1/r|t)u(r) - u(\bar{r})) - (1 - t)u(R)] f(t) dt. \end{aligned} \quad (50)$$

As in the proof of Proposition 1, we can show that

$$\lim_{r \searrow \bar{r}} \frac{1}{r - \bar{r}} \int_{1/r}^{\bar{t}} [t(\alpha_i(1/r|t)u(r) - u(\bar{r})) - (1 - t)u(R)] f(t) dt = 0, \quad (51)$$

and that

$$\begin{aligned} \lim_{r \searrow \bar{r}} \int_0^{1/r} \left[t \frac{u(r) - u(\bar{r})}{r - \bar{r}} + (1-t) \frac{u \circ g(r) - u \circ g(\bar{r})}{r - \bar{r}} \right] f(t) dt = \\ \int_0^{1/\bar{r}} \left[u'(\bar{r}) - Ru' \left(\frac{R(1-t\bar{r})}{1-t} \right) \right] f(t) dt. \end{aligned} \quad (52)$$

Since $u'(\bar{r}) > Ru' \left(\frac{R(1-t\bar{r})}{1-t} \right)$ whenever $t < \bar{t}$, we conclude that

$$\lim_{r \searrow \bar{r}} \frac{U(r) - U(\bar{r})}{r - \bar{r}} > 0. \quad (53)$$

The proof is now complete. ■

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